Week 8 - Friday

COMP 2100

Last time

- What did we talk about last time?
- 2-3 and red-black tree practice
- AVL trees
- Balancing trees by construction

Questions?

Project 2

Assignment 4

Call for Mentors

- Hanby Elementary in Westerville is looking for mentors for two programs
- FIRST LEGO League (Tuesdays 2:45–4:45 PM)
  - Students research real-world problems and build and program LEGO EV3 Mindstorm robots to complete themed missions.
- Girls Who Code (Mondays 2:45–3:45 PM)
  - Girls Who Code students work on projects such as app or game design, website creation, and 3D printing prototypes that solve real-world problems.
- Both are great opportunities to give back to the community and build your resume
- If interested, send me an e-mail

Hash Tables

Recall: Symbol table ADT

- We can define a symbol table ADT with a few essential operations:
  - put(Key key, Value value)
    - Put the key-value pair into the table
  - get(Key key):
    - Retrieve the value associated with key
  - delete(Key key)
    - Remove the value associated with key
  - contains(Key key)
    - See if the table contains a key
  - isEmpty()
  - size()
- It's also useful to be able to iterate over all keys

Unordered symbol table

- We have been talking a lot about trees and other ways to keep *ordered* symbol tables
- Ordered symbol tables are great, but we may not always need that ordering
- Keeping an unordered symbol table might allow us to improve our running time

Hash tables: motivation

- Balanced binary search trees give us:
  - $\Theta(\log \boldsymbol{n})$ time to find a key
  - $\Theta(\log \boldsymbol{n})$ time to do insertions and deletions

- Can we do better?
- What about:
  - $\Theta(1)$ time to find a key
  - $\Theta(1)$ to do an insertion or a deletion

Hash tables: theory

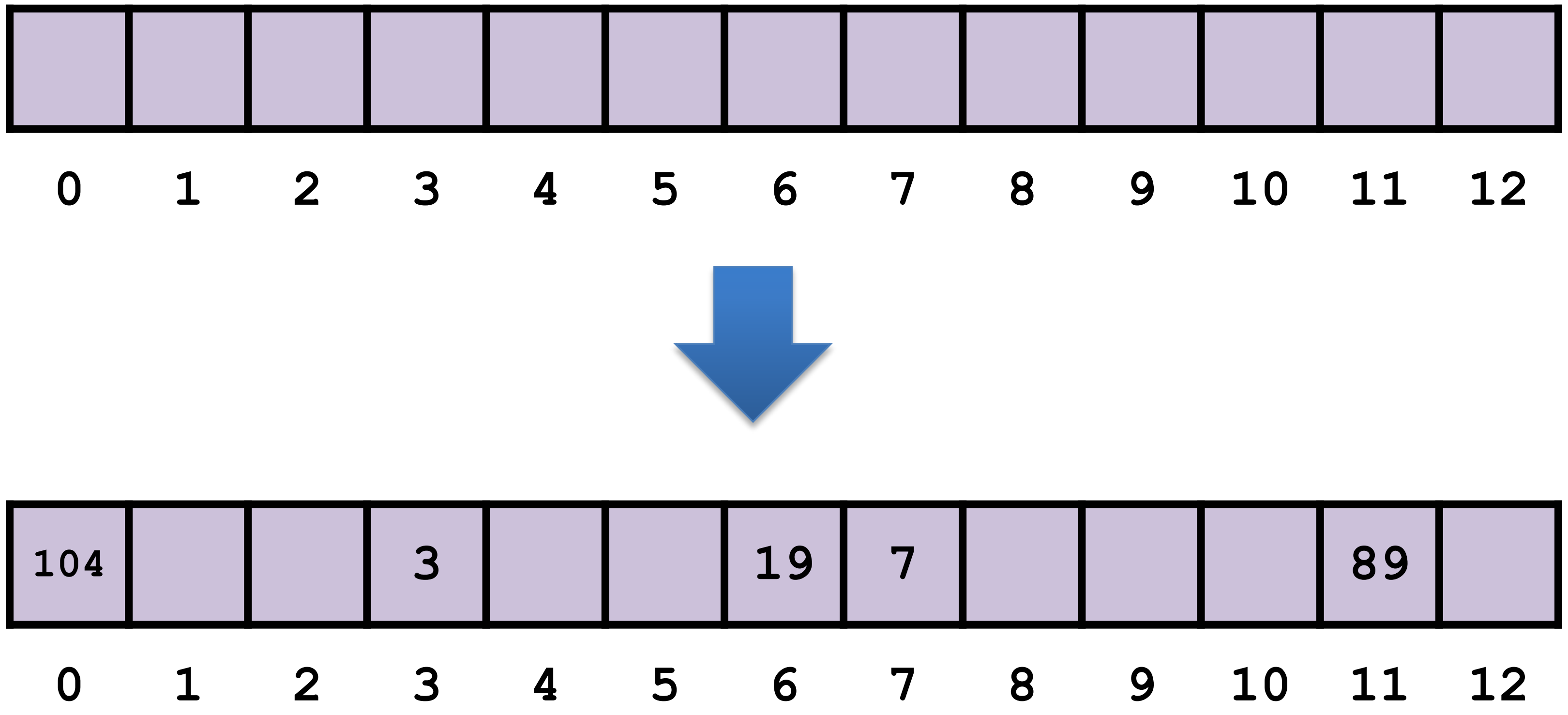
- We make a huge array, so big that we'll have more spaces in the array than we expect data values
- We use a **hashing function** that maps keys to indexes in the array
- Using the hashing function, we know where to put each key but also where to look for a particular key

Hash table: example

- Let's make a hash table to store integer keys
- Our hash table will be 13 elements long
- Our hashing function will be simply modding each value by 13

Hash table: example

- Insert these keys: 3, 19, 7, 104, 89



Hash table: example

- Find these keys:

0	1	2	3	4	5	6	7	8	9	10	11	12
104			3			19	7				89	

- 19
  - YES!
- 88
  - NO!
- 16
  - NO!

Hash table: issues

- We are using a hash table for a space/time tradeoff
- Lots of space means we can get down to $\Theta(1)$
- How much space do we need?
- How do we pick a good hashing function?
- What happens if two values **collide** (map to the same location)

Example

- Determine if a string has any duplicate characters
- Weak!
- Okay, but do it in O(***m***) time where ***m*** is the length of the string

Hash Functions

What are we looking for?

- We want a function that will map data to buckets in our hash table
- Important characteristics:
  - **Efficient**: It must be quick to execute
  - **Deterministic**: The same data must always map to the same bucket
  - **Uniform**: Data should be mapped evenly across all buckets

Division

- We want a function $h(k)$ that computes a hash for every key k
- The simplest way of guaranteeing that we hash only into legal locations is by setting $h(k)$ to be:
- $h(k) = k \bmod N$ where N is the size of the hash table
- To avoid crowding the low indexes, N should be prime
- If it is not feasible for N to be prime, we can add another step using a prime $p > N$:
- $h(k) = (k \bmod p) \bmod N$

Division Pros and Cons

- Pros
  - Simple
  - Fast
  - Easy to do
  - Good if you know nothing about the data
- Cons
  - Prime numbers are involved (What's the nearest prime to the size you want?)
  - Uses no information about the data
  - If the data is strangely structured (multiples of ***p***, for example) it could all hash to the same location

Folding

- Break the key into parts and combine those parts
- **Shift folding** puts the parts together without transformations
  - SSN: 123-45-6789 is broken up and summed 123 + 456 + 789 = 1,368, then modded by ***N***, probably
- **Boundary folding** puts the parts together reversing every other part of the key
  - SSN: 123-45-6789 is broken up and summed 123 + **654** + 789 = 1,566, then modded by ***N***, probably

Folding Pros and Cons

- Pros
  - Relatively Simple and Fast
  - Mixes up the data more than division
  - Points out a way to turn strings or other non-integer data into an integer that can be hashed
  - Transforms the numbers so that patterns in the data are likely to be removed
- Cons
  - Primes are still involved
  - Uses no special information about the data

Mid-Square Function

- Square the key, then take the "middle" numbers out of the result
- Example: key = 3,121 then $3{,}121^2$ = 9,740,641 and the hash value is 406
- One nice thing about this method is that we can make the table size be a power of 2
- Then, we can take the $\log_2 \boldsymbol{N}$ middle bits out of the squared value using bitwise shifts and masking

Mid-Square Pros and Cons

- Pros
  - Randomizes the data a lot
  - Fast when implemented correctly
  - Primes are not necessary
- Cons
  - Uses no special information about the data

Extraction

- Remove part of the key, especially if it is useless
- Example:
  - Many SSN numbers for Indianapolis residents begin with 313
  - Removing the first 3 digits will, therefore, not reduce the randomness very much, provided that you are looking at a list of SSNs for Indianapolis residents

Extraction Pros and Cons

- Pros
  - Uses information about the key
  - Can be efficient and easy to implement
- Cons
  - Requires special knowledge
  - Careless extraction of digits can give poor hashing performance

Radix Transformation

- Change the number to a different base
- Then, treat the base as if it were still base 10 and use the division method
- Example: 345 is 423 in base 9
- If ***N*** = 100, we could take the mod and put 345 in location 23

Radix Transformation Pros and Cons

- Pros
  - If many numbers have similar final digits or values mod ***N*** (or ***p***), they can be randomized by this method
- Cons
  - Choice of base can be difficult
  - Effects are unpredictable
  - Not as quick as many of the other methods
  - Values that didn't collide before might now collide

Collisions

The real problem with hash tables

- What happens when you go to put a value in a bucket and one is already there?
- There are a couple basic strategies:
  - Open addressing
  - Chaining
- **Load factor** is the number of items divided by the number of buckets
  - 0 is an empty hash table
  - 0.5 is a half full hash table
  - 1 is a completely full hash table

Open addressing

- With open addressing, we look for some empty spot in the hash table to put the item
- There are a few common strategies
  - Linear probing
  - Quadratic probing
  - Double hashing

Linear probing

- With linear probing, you add a step size until you reach an empty location or visit the entire hash table
- Let ***h***(***k***) be the initial hash function
- ***h***(***k***,***i***) = ***h***(***k***) + ***ci***, for ***i*** = 0, 1, 2, 3…

104			3			19	7				89	
0	1	2	3	4	5	6	7	8	9	10	11	12

- Example: Add 6 with a step size of 5

104			3			19	7	6			89	
0	1	2	3	4	5	6	7	8	9	10	11	12

Quadratic probing

- For quadratic probing, use a quadratic function to try new locations:
- $h(k,i) = h(k) + c_1 i + c_2 i^2$, for $i = 0, 1, 2, 3\ldots$

104			3			19	7				89	
0	1	2	3	4	5	6	7	8	9	10	11	12

- Example: Add 6 with $c_1 = 0$ and $c_2 = 1$

104			3			19	7			6	89	
0	1	2	3	4	5	6	7	8	9	10	11	12

Double hashing

- For double hashing, do linear probing, but with a step size dependent on the data:
- $\boldsymbol{h}(\boldsymbol{k},\boldsymbol{i}) = \boldsymbol{h}_1(\boldsymbol{k}) + \boldsymbol{i}\cdot\boldsymbol{h}_2(\boldsymbol{k})$, for $\boldsymbol{i} = 0, 1, 2, 3\ldots$

104			3			19	7				89	
0	1	2	3	4	5	6	7	8	9	10	11	12

- Example: Add 6 with $\boldsymbol{h}_2(\boldsymbol{k}) = (\boldsymbol{k} \bmod 7) + 1$

104	6		3			19	7				89	
0	1	2	3	4	5	6	7	8	9	10	11	12

Open addressing pros and cons

- Open addressing schemes are fast and relatively simple
- Linear and quadratic probing can have clustering problems
  - One collision means more are likely to happen
- Double hashing has poor data locality
- It is impossible to have more items than there are buckets
- Performance degrades seriously with load factors over 0.7

Chaining

- Make each hash table entry a linked list
- If you want to insert something at a location, simply insert it into the linked list
- This is the most common kind of hash table
- Chaining can behave well even if the load factor is greater than 1
- Chaining is sensitive to bad hash functions
    - No advantage if every item is hashed to the same location

Deletion

- Deletion can be a huge problem
- Easy for chaining
- Highly non-trivial for open addressing
- Consider our linear probing example with a step size of 5

104			3			19	7	6			89	
0	1	2	3	4	5	6	7	8	9	10	11	12

- Delete 19
- Now see if 6 exists

Perfect Hash Functions

- If you know all the values you are going to see ahead of time, it is possible to create a minimal perfect hash function
- A minimal perfect hash function will hash every value without collisions and fill your hash table
- Cichelli's method and the FHCD algorithm are two ways to do it
- Both are complex
- Look them up if you find yourself in this situation

Upcoming

Next time…

- Implementing hash tables
- **`Map`** in the JCF
  - **`HashMap`**
  - **`TreeMap`**
- Introduction to graphs

Reminders

- **Finish Project 2**
  - **Due tonight by midnight!**
- Start Assignment 4
- Keep reading 3.4
- Read 4.1
- **No class on Monday!**